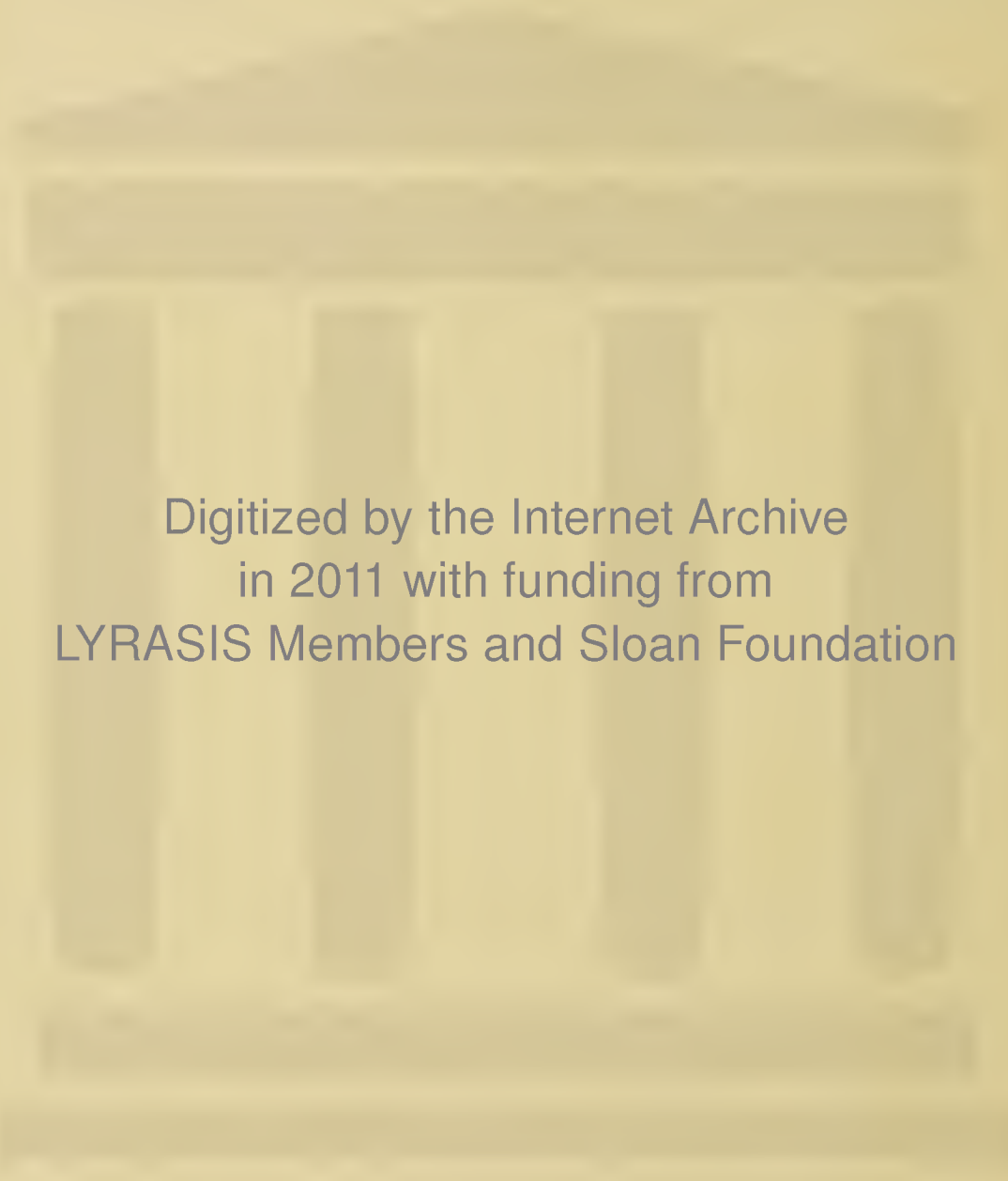
THE PLACE OF MATHEMATICS IN THE HIGH SCHOOLS OF HIGHLANDS COUNTY, FLORIDA

MARY CORDELIA SHEARER



Approved
5/28/49 Edward H. Hennessy Jr.

THE PLACE OF MATHEMATICS IN THE HIGH SCHOOLS

OF

HIGHLANDS COUNTY, FLORIDA

An Analysis of the Mathematics
Program in the Three High Schools

MARY CORDELIA SHEARER

Submitted in partial fulfillment of the
requirements for the degree of Master of
Arts in the Graduate School of
Florida Southern College

1940

PREFACE

The writer's interest in this problem has been aroused by comments made by pupils after a standardized test had been administered. Results were being discussed privately with freshmen, sophomores, juniors, and seniors. A freshman who is taking general mathematics asked, " What mathematics can I take next year?" A sophomore who is taking no courses in mathematics remarked, " I wish I knew more arithmetic. I am low in mathematics on every test." A junior who has taken no mathematics since his freshman year inquired, " Is there any way to get mathematics in my senior year? " A senior who has elected no such courses said, " I wish now that I had taken mathematics. I want to get a job after graduation, and I am beginning to get scared that I might be asked to do some figuring".

The writer acknowledges with gratitude the helpful suggestions of Dr. B.P.Reinsch. She also expresses deep appreciation to Professor Edward Lee Fleming for patiently directing this thesis.

TABLE OF CONTENTS

LIST OF TABLES

CHAPTER I

THE PROBLEM

GENERAL STATEMENT

It is the purpose of this study to present evidence of the need of revision of the curriculum of Highlands County schools that will result in the desired mastery of mathematical skills and concepts and in the carry over of such skills and concepts into life situations.

SPECIFIC PROBLEMS

1. What percentage of the pupils of Highlands County are enrolled in elective mathematics courses in grades ten, eleven, and twelve?

2. What method is used to present mathematics to the attention of the students at the end of each year before enrolling them for the next year's work?

3. Who takes general mathematics, and who algebra in grade nine?

4. What relationship exists between the intelligence quotient and mathematical ability as shown by tests given in October, 1948?

5. What progress is made in six months as evidenced by tests administered in April, 1949?

6. What common errors are made in applying skills?

7. What are the requirements for mathematics in Highlands County?

8. What mathematics is necessary for a useful citizen?

9. What is the responsibility of the school in regard to the subject of mathematics?

10. Has the mathematics teacher a special role in the mathematics program of the school?

DEFINITION OF TERMS

"Administration" is understood to mean county superintendent, county supervisor, and principals of high schools.

"Requirements" shall be considered the mathematics credits required for graduation from high school.

"Faculties" refers to teaching personnel.

"Mathematics Curriculum " means the mathematics offerings in the school program of the county.

DELIMITATIONS

This investigation covers grades nine, ten, eleven, and twelve in the three high schools of Highlands County.

BASIC ASSUMPTIONS

Considering that fifty per cent of students take no mathematics courses during the last three years of high school, as shown by the county survey made in this investigation, it is assumed that fully half of those who graduate are deficient in mathematics; e.g., not equipped to purchase thriftily, to compute wages correctly, nor to adequately fill positions requiring ordinary mathematical skills.

BASIC HYPOTHESIS

A thorough study of the mathematics situation in the county may lead to an aroused interest that will result in more intelligent guidance to mathematics courses, additional mathematics requirements, and an enriched curriculum.

THE NEED FOR THE STUDY

The pressure to make mathematics easier for students is inclined to take the form of making it less meaningful. Zant says[1] that ten to twenty per cent of the schools of the nation have business arithmetic as an elective, and a few have remedial arithmetic classes. He states that it is the

---

1. James H. Zant, " The Improvement of High School Mathematics Courses as Recommended by the Commission on Post War Plans," Mathematics Teacher. October, 1946. p.269.

responsibility of the school to

1. Provide sound mathematical training for future leaders in science, mathematics, and other learned fields, and to

2. Insure mathematical competence for the ordinary affairs of life for all citizens as a part of general education.

Carl G. Norberg, Department of Mathematics, Greenbrier Military School, Lewisburg, West Virginia, states that mathematics is second to no other subject in high school.

INCIDENCE OF THE PROBLEM

In 1944 four seniors in the high school at Avon Park, Florida, asked for solid geometry and trigonometry. No class could be organized with less than ten members. The four had to go to college without the desired courses. They were forced to spend valuable time there studying these high school subjects. The same situation arose in 1948, in the same school when five junior boys asked for plane geometry. Equally deplorable is the position of those whose terminal mathematics course consists of reading and writing numbers, simple graphs and scale drawings, common and decimal fractions, and elementary accounting because all courses beyond grade nine are elective. Such courses are chosen only by those pupils who are ambitious.

HISTORICAL BACKGROUND OF THE PROBLEM

The first arithmetic textbook written and printed in

the United States appeared in Boston in 1729 but was used very little because stress was not placed on arithmetic at that time, the subject being frequently omitted altogether.

A century later practical demands for arithmetic arose, and it was placed in the school curriculum. Warren Colburn prepared a text containing no rules, introducing drill and oral instruction, and using practical problems. This book had a greater sale than any other elementary arithmetic textbook ever had.

About the time of the Civil War, emphasis was placed on the disciplinary value of arithmetic, and the texts contained an over-abundance of material for instruction. An attempt to eliminate material led to the first result of scientific research applied to arithmetic, and the Report of the Committee of Fifteen, 1895, pointed to a practical emphasis on the subject. At present the chief objective is to train the pupil to become an intelligent member of his community, able to deal masterfully with his personal affairs and with his share of community enterprise. The level of mathematical literacy required for these purposes is continually rising. Men now measure the change in the rotation rate of the earth, a rhythmic increase or decrease over a range of forty seconds in eighty years. The increasing refinement of measurement in industry, as well as

the extended use of new types of maps, is typical of the increased precision needed in dealing with social problems.

Betz thinks[1] that mathematics is being assigned to the scrap heap in our high schools. Lela Van Engen, University of Wyoming, Laramie, Wyoming, testifies that many adults struggle with life situations due to lack of understanding of arithmetic. It is the opinion of H.G.Wells[2] that the time may not be too remote when it will be understood that for complete initiation as an efficient citizen, it will be necessary to be able to compute and to think in averages and maxima and minima, as it is now understood to be necessary to be able to read and to write. Schlauch, in his autobiography, writes:

> I am convinced that in every field of human endeavor mathematics can be employed to make better adjustments, greater control, and less waste.[3]

The Harvard University Committee on General Education in a Free Society advises[4] that it would be very much better if even the minimum program of mathematics instruction for

---

1. W. Betz, " Looking Again at the Mathematical Situation", Mathematics Teacher. December, 1948. p.373.
2. H.G.Wells, Mankind in the making. as quoted by Leo J. Brueckner, " The Social Phase of Arithmetic Instruction", National council of Teachers of Mathematics, Sixteenth Yearbook. 1941. p. 156.
3. Wm.S.Schlauch, " An Autobiography", Mathematics Teacher. November, 1948. p. 301.
4. Harvard University Committee on General Education in a Free Society. 1946. p.166.

the pre-college student were distributed over the entire secondary school course. Many educators recommend a general education course, including basic mathematics, for all seniors who have not elected mathematics courses in the sophomore and junior years of high school.

ORGANIZATION OF REMAINDER OF STUDY

The next chapter attempts to evaluate mathematics as a study for high school pupils. The third discusses the double track plan used in Florida schools for grade nine. The fourth points out the relationship between the I.Q. and mathematic efficiency and notes the revival of interest in the subject since World War II. The fifth makes an effort to show the progress made by the pupils of Highlands County between October and May of the school year 1948-1949. The sixth lists common errors discovered by the writer in her classes, and suggests causes and remedies. The seventh shows the mathematics requirements in the state of Florida as revealed by answers to a questionnaire sent to the sixty-seven counties. The eighth enumerates mathematics requirements for citizenship. The ninth places responsibility for mathematical training upon the school. The tenth gives the teacher the key role in the program. The last chapter lists some of the outcomes which the writer hopes may result from this study.

CHAPTER II

AN EVALUATION OF MATHEMATICS

Mathematics is the oldest of all the sciences. Professor Hogben says[1] that the history of mathematics is the ' mirror of civilization'. Mathematics sprang from problems of feeding, clothing, and shelter, and throughout the world of today is regarded as indispensable for further progress. It is growing so rapidly that the present has been described as the Golden Age of Mathematics. Mathematicians regard this science as one of man's most appropriate activities.

Betz likens[2] mathematics to a tree, the roots being the human needs out of which mathematics grew. The subject is the product of racial experiences, the joint contribution of the ages. It is a system of ideas that exists and grows only in the mind. The trunk of the tree is the framework of mathematics, including basic concepts, principles, facts,

---

1. Lancelot Hogben, Mathematics for the Million W.W.Norton and Company, 1937. p.38.
2. William Betz, " Competence in Mathematics", Mathematics Teacher. May, 1948. p.200.

tested rules of computation, established methods of solving problems, and its modes of thinking. This skeleton structure has taken the form of a system embracing arithmetic, geometry, algebra, and trigonometry. The branches of the tree are mathematical applications. There cannot be branches without the tree; therefore mathematics cannot be competently applied without regard for the basic ideas and techniques. Expansion of industry, of technology, or commerce, and of mathematical research causes the tree to grow.

The ideas, processes, and applications of arithmetic as we have it today form a highly perfected, perfervid, permanent, and universal social heritage. Various practical arts such as making bows and arrows and dividing pieces of land, forced upon men in the struggle to survive and raised later to a higher plane by his desire to make living more comfortable and refined, prepared the way for geometry.

Such concepts as equality, congruence, similarity, symmetry, ratio and proportion, woven so firmly into geometry, also contributed, under the spur of problems of indirect measurement, to the development of trigonometry which the Greeks had used in astronomy.

The problems that were being subjected to mathematical analysis were constantly becoming more intricate and revealed the necessity for methods more general and techniques more powerful and universal than the old ones. Out of this need

came algebra, a subject which has shown a disposition to crowd into more and more places with willingness to take upon itself a larger amount of the work of the world. A fusion of algebra and geometry produced analytic geometry.[1]

The purpose of systematic learning in arithmetic, according to McConnell[2], is to provide pupils with methods of thinking, with ideas of procedure, with meaning inherent in number relations, and with general principles of combination and arrangement in order that the quantative situations of life may be handled intelligently. Buswell states[3] that the mastery of mathematical relationships is essential in making many adjustments which eventuate in desirable personality traits.

Many people discover a need for mathematics after leaving school. At Scapa Flow, Hooper[4] coached Royal Air Force men who were anxious to qualify as air crew candidates. His object was to enable persons of average intelligence to achieve a clear grasp of the essentials of arithmetic, algebra, geometry, and trigonometry with

---

1. The Joint Commission of the National Council of Teachers of Mathematics and the American Mathematical Association, " The Place of Mathematics in Secondary Education, Fifteenth Yearbook. 1940. p. 157.
2. T.R.McConnell, "Recent Trends in Learning Theory," National Council of Teachers of Mathematics, Sixteenth Yearbook. p.270.
3. Guy T. Buswell, "Function of Subject Matter in Relation to Personality," Sixteenth Yearbook. p.14 1941.
4. A.Hooper, A Mathematics Refresher. p.1. 1946.

the minimum expenditure of time and effort. The first essential was to break down barriers which prevent so many intelligent people from mastering the beautiful simplicity and ordered sequence of mathematical processes. Reasoning mathematically is like climbing up a mental ladder; one rung leads to the next. The feet must be firmly set on each rung before an attempt is made to go higher.

There are those who go through life with feelings of inferiority and dread simply because they have not learned enough mathematics to enable them to behave intelligently in mathematical situations. Many women, when they find it necessary to triple a recipe, measure out two-thirds of a cup of sugar three times rather than hazard the mental operation of "How much is three times two-thirds?" innumerable people are caught in the trap of installment buying because they do not know how to work out the simple arithmetic of the situation. Short-term loan agencies flourish on the patronage of people who never realize the exorbitant rates of interest they are paying until they are trapped by the impossibility of keeping pace with the rapidly accumulating size of their loans.

Snedden says:

> The position of mathematics in the Tree of Knowledge is a challenge to teachers of mathematics, as well as an assertion of the value of the subject. It is a challenge to pupils,

> showing them not only what they need in order to succeed, but what they must know if they are to comprehend certain essential elements of the civilization they are to share.[1]

Progress in all the sciences depends upon mathematics. The student who drops high school mathematics drops medicine, engineering, chemistry, biology, physics, economics, social science, and psychology.

Mathematical methods set up an ideal, a norm, a pattern which serves as a relentless spur to improvement. Mathematics succeeds because it searches tirelessly for the principles that underlie a situation or the weak point in a problem. It analyzes, it dissects, it relates; it tends to eliminate what is irrelevant or superfluous in order to attain an economy in thinking and expression. In the evolution of science and technology mathematics has constantly furnished essential and valuable aid.

Mathematics has the widest scope and most pervasive influence upon material civilization and the practical affairs of human life of any known subject. It is the structural steel foundation on which material civilization rests. It is the hand-maiden of commerce, the guiding spirit of engineering, the practical partner in architecture.

The Empire State Building, the Roosevelt Dam, the

---

1. D.S.Snedden, Educational Sociology for Beginners. p. 52. 1928.

> Panama Canal could not stand without the directive guidance and protective machinery of mathematics. Cities spring up because matnematicians quietly sit at desks and determine how the magic is to be wrought. There is no whit of exaggeration in the assertion that, without mathematics, material civilization would collapse.[1]

Mathematics must be studied for some time before it pays extensive returns. One lesson in first aid may enable one to save a life. Not so in the field of mathematics. Comparatively few pupils in the secondary school know what their later activities and studies will be. On entering college, they find doors closed to desired fields because of lack of mathematical preparation. The subject is so extensive and so difficult, requiring systematic and protracted study, as to be unsuitable for the general application of the doctrine of postponement or incidental learning. Keyser states:

> From time immemorial, there has been but one way to become a mathematician and there will never be another; it is the way interior to the subject and involves years of assiduous toil[2]

Fry points out[3] that mathematics is a language which simplifies the process of thinking and makes it more

---

1. William Allen Neilson, Roads to Knowledge. p.324. 1937.
2. Cassius J.Keyser, Mathematical Philosophy. p.148. 1922.
3. Thornton C. Fry, Research -- A National Resource p. 275 as quoted by Final Report of the Commission on Post War Plans of the National Council of Teachers of Mathematics, Guidance Pamphlet in Mathematics for High School Students. p.18. 1947.

reliable. This, he points out, is its principal service to industry. He then lists ways in which mathematics is useful to industry as follows:

1. It provides a basis for interpreting data in terms of a preconceived theory, thus making it possible to draw deductions from them regarding things which could not be observed conveniently, if at all.
2. When data are incompatible with the preconceived theory, a mathematical study frequently aids in perfecting the theory itself.
3. Mathematics frequently aids in promoting economy either by reducing the amount of experimentation required, or by replacing it entirely.
4. Sometimes experiments are virtually impossible and mathematics must fill the breach.
5. Mathematics frequently plays an important part in reducing complicated theoretical results and complicated methods of calculation to readily available working form.

The field of mathematics holds opportunities for women as well as men. There is a big field for teaching mathematics in high schools and colleges. The competent woman is sought also for computational and statistical work with insurance and business firms, government agencies, and industrial concerns.

Mathematics is a subject second to none in its natural interest, in its possibilities for material exhibition, in its multiplied intimate connections with every phase of life, and in its dramatic human relationships. It is a subject excelled by none in the possibility for complete satisfaction of the human desire to understand meanings, and to go to the bottom of a question. Outcomes of mathematics

are both utilitarian and appreciation. The utilitarian aims are those which make the individual more effective personally and socially. Appreciation aims make the world more interesting to the pupil and the pupil more interesting to the world.

During the past generation, schools have experimented with "activity curricula" and with"experience curricula" in an effort to make education more meaningful and pleasurable for the child. Since these curricula were readily adaptable to social studies and language arts, these areas were frequently emphasized. The net result has been that arithmetic has suffered, and general achievement in the program of mathematics is not now satisfactory.

If mathematics is to be given prominence in education, every effort should be made to reduce the number of teachers who feel incompetent to teach arithmetic in the grades, and who teach it with unhappiness and frustration. There is need for better courses of study and for better trained teachers.

Mathematics should not be forced upon pupils who have a rebellious distaste for the subject. However, in view of the increasing importance of mathematics to civilization because of the techniques it has perfected as well as its methods of reasoning, ample opportunity should be given to all individuals to continue their mathematical training as far as their powers allow and as other conditions permit.

Mathematics, then, should be a permanent ingredient

of every balanced school curriculum. Snedden expresses the following views:

> The teaching given us in the fields of physics, engineering, surveying, and astronomy was surely never designed to make astronomers, African explorers, writers of poetry, or painters of pictures out of us. But we are cultured persons to the extent that we have rich appreciations precipitated from such long range contacts as we were able to make in these great human enterprises.[1]

The teaching of mathematics may help pupils to keep their accounts better or get a job after graduation, but if it develops in them an understanding of number relations, if it teaches them to visualize distances and quantities, it is training them culturally, and they will forever after be more sensitive, more appreciative, and more understanding.

Too many pupils take no mathematics in high school. Too many administrators, supervisors, home room teachers, and other guidance personnel fail to realize the large number of persons needed to do the scientific, professional, or highly technical tasks of modern life. Schorling says[2] that because of the present emphasis on technical lines of work, more and more professions are being touched by the necessity for the use of mathematics. He adds that many

---

1. D.S.Snedden, Educational Sociology for Beginners. p.135.1928.
2. Raleigh Schorling, " Guidance, Mathematics Teacher. January, 1949. p.26.

present day high school pupils will want to become surgeons, actuaries, statisticians, surveyors, or scientific researchers. These include a goodly fraction of the high school population. Their rigorous training should not be neglected.

The poor student has a right to an education that will not frustrate him. Many seemingly unpromising pupils have developed into bright intellectuals. The talented student has a right to an education which will develop his talents as quickly as possible. The writer believes that neglecting to train the gifted for leadership constitutes one of the greatest weaknesses of the high school system.

Educational objectives will center around three permanent factors, namely, the physical universe, society, and the child. Mathematics makes outstanding contributions to the attainment of these objectives. It is for the good of society that each year there should go forth from the secondary schools a large number of young people with marked proficiency in the technical skills of mathematics, together with a fundamental understanding of some of its concepts. The steady flow of such a group into our democracy is the responsibility of school administrators.

We have not been sagacious enough in the use of human resources as regards early identification of children with a flair for mathematics and their proper culture.

There should be some way of identifying, early in school life, youth who have special talent. These young people should surely have proper nurture through school, college, and university. There must be some way to give an education to those who want it and to persuade those who are competent to want it.

Mathematics teachers should acquaint pupils with persons, groups, and institutions using mathematics in furthering the advance of human knowledge or in solving community and social problems. To a rapidly increasing extent specialists are being called for in all lines of adult enterprise, and with specialization comes mathematization of all fields. Business administration, psychology, education, biology, all invoke mathematical statistics of an advanced sort.

Before the close of school, pupils are asked to register for the following year's courses in Highlands County. It is customary for the principal to suggest to the students that mathematics and science are good subjects to enroll in, especially for those who plan to go to college. Too often pupils ask other pupils if algebra is hard or if they liked geometry, and determine their courses accordingly. Thus many sophomores, juniors, and

seniors have no mathematical instruction. The following table, page twenty, shows the number and the percentage of pupils in Highlands County high schools who are taking mathematics courses in the school year 1948-1949.

TABLE I

NUMBER AND PERCENTAGES OF PUPILS IN THE HIGH SCHOOLS OF HIGHLANDS COUNTY IN GRADES X, XI, AND XII WHO TAKE MATHEMATICS

GRADE	ENROLLMENT	NUMBER	PER CENT TAKING
X	117	30	27
XI	126	57	46
XII	107	42	39
TOTALS	350	129	37

CHAPTER III

EITHER GENERAL MATHEMATICS OR ALGEBRA FOR GRADE NINE

As the world has grown, so the work with numbers has grown. When the world has faced the mysteries of the universe, numbers have assisted in solving its problems. When commerce and science have shown new needs in computation, arithmetic has always been ready to lend a hand.

Mathematics is an essential language for the description of relationships and changes. Galileo found it necessary to have a method for expressing relationship between time and distance. The language of the machinist today involves measurement, formulas, equations, and the relationship between operations. The chemist, physicist, biologist, radio operator, meteorologist, aviator, and electrician have need of the language which mathematics has developed in the last thousand years. They must have its methods of thought and its clarity in expressing relationships.

The Florida State Department of Education feels[1] that every pupil in the secondary school is entitled to an

---

1. Florida Department of Education, Suggestions for Guidance in Florida Schools, Bulletin Number 19.p.9. 1947.

opportunity early in his high school career to project for himself, under guidance, a program of studies for his high school years. The Florida Citizens Committee survey in 1947 revealed that many schools in Florida have no systematic plan for such guidance. They offer a double track curriculum for grade nine, general mathematics and algebra. The quicker pupils take algebra; those who have been considered slow in arithmetic take general mathematics, and for many of them this is their terminal year in the subject.

Peavy says[1] there are three chief reasons why junior high school pupils are slow:

1. Families move about and change schools often.
2. Attendance is irregular because of illness or parental indifference.
3. Some pupils are not mentally equipped to do first rate work.

For a long period of time mathematics was a neglected subject. With the coming of World War II, more and more people found that they must improve their knowledge of mathematical procedure. The military authorities demanded greater knowledge of mathematical principles, greater ability to put figures to work on the part of those who were being inducted into the armed forces. Operators of defense plants found that those employees who had a thorough understanding of mathematics were of the most value to them.

---

1. Katherine B.Peavy, " Arithmetic and Life Experiences", Grade Teacher.March, 1948. p.94.

Late in 1944 Allied victories directed thought to the post-war period which would bring with it new problems. Government and private industry alike began to think along the lines of a rehabilitation program. In this scheme mathematical ability was regarded as a prime individual asset, and, for the untrained plans to supply it were made.

The need for trained men and women, especially for those with various degrees of training in mathematics and science, is still critical. It is the obligation of the high school to see that pupils have thorough training in functional mathematics before they enter the service of their country. It is the duty of the government to see that they are able to continue it after they leave the service. Many boys enlist from grade nine or ten.

Perry believes[1] that the study of physical science, and therefore the study of mathematics, by everybody, however poor or however rich, is of the utmost importance to our country, not merely for the knowledge it imparts, but for producing the scientific habit of thought, giving to every unit of population the power to think for itself, and so producing the greatest happiness and giving the

---

1. John Perry, " Paper Read at the General Session of Science and Mathematics Teachers, Indianapolis, Indiana, November 26, 1902, as quoted by W.D.Reeve, " General Mathematics for Grade Nine,"School Science and Mathematics. February, 1949.p.100.

greatest strength to all the nation. Thomas Edison once said[1] that after examining 10,000 college graduates, he had come to the conclusion that thinking is unnatural. The average student often avoids everything that requires him to use his power of thinking.

There are two unfortunate attitudes in the schools of today in the state of Florida. The first is that general mathematics is for dull normals. The writer asked her general mathematics pupils in the first session of the year why they were taking general mathematics. More than half of the members of that class wrote, " Because I'm too dumb to take algebra". A good general mathematics course includes much that is good for bright pupils. The second unhappy attitude is that one year of mathematics in the school after grade eight is sufficient. The system of social promotion used in the state of Florida puts some pupils into grade nine classes who are on mathematics levels from grade three to grade six. The best that can be hoped for them in one year of training in mathematics is a knowledge of fractions, decimals, and areas and volumes of simple figures. They then enter high school.

---

1. Lurline Stewart, " The Blight of Mathematical Slang," Mathematics Teacher. February, 1949.p.99.

One of the greatest weaknesses in the high school is that too many pupils take no mathematics and later find that they lack specific training for what they want to do. When a student does not seem to have the ability to succeed in algebra, some schools advise him to take no mathematics beyond grade eight. Pupils should be required to take courses designed to give an opportunity to acquire the necessary competences which they lack.

Who should take algebra? The one who can do a good piece of work in algebra without too much grief to himself, to his family, and to the teacher; and the one who enjoys algebra and is likely to make some future use of it. Pupils might well be told that few adults, if any, who are competent in algebra have ever regretted time spent in learning it; that algebra is an easy subject if a student comes to it with adequate training, and if it is well taught, the structure being built carefully step by step; and that there are few subjects which pupils like better than algebra if it is well taught.

There is a general belief that a good foundation in arithmetic is essential for preparation for algebra, but this question could stand investigation. The pupil who has done good work in arithmetic will very likely do good work in algebra.

Butler and Wren explain[1] algebra thus:

> In contrast to arithmetic, algebra is more concerned with the conscious examination and study of processes than with particular answers to particular problems. It is characterized by a much higher degree of generalization and abstraction.

This description sets a high standard for any class in algebra, and many schools may feel that only a small per cent of their pupils could do the work that it implies. The pupil who is unable to generalize will probably not succeed in algebra.

A few schools in Florida offer geometry after first year algebra. Nyberg wishes[2] that more schools had the courage to enroll grade ten pupils in geometry rather than in algebra, after they have had mathematics in grade nine. He says these pupils are not likely to reach any position in industry in which a knowledge or ignorance of either algebra or geometry will alter their lives or influence the national welfare. He says that the geometry can be made more interesting than algebra because there are figures and models for illustrations, and geometry also is less abstract.

Reeve proposes[3] that mathematics be arranged so that we shall not have courses in algebra, geometry, or

---

1. Charles Henry Butler and F.L.Wren, The Teaching of Secondary Mathematics. p. 268. 1941.
2. Joseph A.Nyberg,"Notes From a Mathematics Classroom," School Science and Mathematics. January, 1948.p.56.
3. W.D.Reeve, "Mathematics for Grades Nine through Twelve," School Science and Mathematics. February, 1949. p.100.

trigonometry as such, but a definitely arranged and psychologically ordered course in mathematics. This plan would give the pupil who desires it a four year course in mathematics in the secondary school. At the end of such a course, he would be as well or better prepared than most college freshmen, for the course would include arithmetic, algebra, plane and solid geometry, analytic geometry, trigonometry, and the leements of calculus.

It is true that our attempt at universal education will bring into the schools many a case of Johnnie Lowique, Winnie Barely-Pass, and Huck Finn who take no stock in mathematics. But boys and girls should be informed of industries that employ mathematics, and they should be led to see that an acquaintance with the subject helps one to live more intelligently in an age as scientific and technical as our own.

Table II, page 28, gives the number of pupils who are taking general mathematics and the number taking algebra in grade nine in the schools of Highlands County, Florida. Grade nine pupils from the entire county enroll in the three schools that have high schools . These three schools are listed separately in the table.

TABLE II

NUMBER AND PERCENTAGES OF PUPILS TAKING GENERAL MATHEMATICS AND ALGEBRA IN GRADE NINE IN THE THREE SCHOOLS OF HIGHLANDS COUNTY, FLORIDA 1948 - 1949

ENROLLMENT	GENERAL MATHEMATICS	PERCENTAGE	ALGEBRA	PERCENTAGE
66	39	60	27	40
58	23	40	35	60
29	29	100	0	0
TOTAL 153	91	66 2/3	62	33 1/3

CHAPTER IV

REVIVAL OF INTEREST

It took a World War to make the American people realize that basic mathematical instruction in the schools of this country was woefully deficient. Admiral Nimitz criticized severely the inadequate preparation in arithmetic of the Navy recruits. There is currently greater emphasis placed on mathematics and greater respect paid to it than formerly.[1]

Carnahan says[2] that mathematics has gained a great deal of respectability since World War II, and that men in service are warning their kid brothers who are now in school to "get Math." He forecasts the following trends for the future:

1. Attention to better pupil understanding will increase.
2. Stress will be placed upon following directions.
3. Ability to estimate will be emphasized.

---

1. W.D.Reeve,"Modern Trends in Mathematics Education," School Science and Mathematics. January, 1948. p.21
2. Walter Carnahan, " Ninth Year Mathematics," Mathematics Teacher. May, 1947. p.83.

4. It will not be deemed necessary to commit to memory very much mathematics.
5. Diagrams will be used more and more to assist in understanding problems.

The degree of mastery of essential mathematical skills and concepts attained by high school students forms an interesting study. Our mathematics offerings seem to serve the fifteen to twenty per cent of pupils who go to college. Outcomes from general mathematics, offered to slow learners and average pupils, should be just as vital as those from sequential courses, given to rapid learners. General mathematics is not a dumping ground nor a makeshift course for weaklings, says Cager[1]. Slow pupils need to be told and retold; topics must be divided into small units for them.

Reason and understanding and not memory or blind imitations are at the basis of effective learning. Maintenance and improvement of the fundamental skills are the chief objectives of the teachers of algebra and geometry. Teachers of general mathematics would do well to have the same objectives. Mathematics is a tool subject, but it is one thing to know what tool to use, and another to know the technique for using the tool, and still another to be able to judge results. Students

---

1. Wm. A. Cager, "Mathematics for the Other Eighty-Five Per Cent," School Science and Mathematics. April, 1949. p.321.

should be able to recognize whether or not their findings are reasonable. They should know what accuracy is, and accuracy should be demanded.

The purpose in studying mathematics is to educate boys and girls to become more self-reliant and to think and act more surely and correctly when they encounter a mathematical situation. Society expects intelligent behavior from its members. A part of this behavior is dependent upon mathematical knowledge. The poverty of the results of arithmetic teaching has been exposed in that children are not able to use the arithmetic they are taught in dealing with the simple quantitative problems of the adult world which they are soon to enter, and in the arithmetical deficiencies of adults who have had eight or more years of school arithmetic, says Brownell.[1]

The question, " What can low I.Q. pupils achieve in the field of mathematics ?" is often asked. During 1947 great interest was manifested in experiments reported by Schmidt who claimed[2] that her work with 254 feeble-minded children in Chicago had raised the I.Q. as much as 19.5 points in three years. This claim was later refuted by

---

1. Wm. A. Brownell, " The Evaluating of Learning in Arithmetic," National Council of Teachers of Mathematics, Sixteenth Yearbook. 1941. p.227.
2. Arthur S. Hill, " Does Special Education Result in Improved Intelligence for the Slow Learner", Journal of Exceptional Children. April, 1948. p.207.

Kirk[1] who went to Chicago and thoroughly investigated that report. The supervisory staff of the DesMoines Department of Pupil Adjustment inaugurated, in 1947, a policy of re-testing pupils who had been assigned to special classes some time previously. The DesMoines data offer little support to a thesis of I.Q. improvement insofar as the effects of adequate schooling upon subnormal or upon borderline children are concerned.

The bi-annual test results show need for individual attention. These tests are not the ultimate criterion of truth, but they do provide a few measures of efficiency, drive, motivation, ambition, skillful use of aptitudes, and desire to achieve. What mathematics should be taught, to whom it should be taught, and how it should be taught will be determined largely by mental tests. Whether ubject contributes appreciably to a student's education frequently depends upon whether the pupil has the intellectual equipment to assimilate it. It also depends upon whether the subject is properly taught. Schorling says[2] that the desired one hundred per cent ideal is obtained by:

---

1. Samuel A. Kirk, " An Evaluation of the Study of Bernardine G. Schmidt," Journal of Exceptional Children. November, 1948. p. 34.
2. Raleigh Schorling, " What's Going on in Your School?" Mathematics Teacher. April, 1948. p.149.

1. Careful analysis by the teacher of methods, used by the pupils
2. Much explanation,
3. A pupil self-directed program of improvement.

Standard achievement tests are given in Highlands County in October and April of each school year. Mental maturity tests are administered every third year. According to Baker[1] pupils having I.Q.'s above 130 are considered gifted; those between 115 and 130 are rapid learners or bright pupils; those between 90 and 115 are normal, and those below 90 are considered slow learners.

The tables which follow are based on the achievement tests given in October, 1948, in the schools of Highlands County. Table III compares the I.Q.'s from 60 to 150 with the achievement scores from 0 to 100. Table IV shows a comparison of the number of boys with the number of girls having achievement scores between 0 and 100.

---

1. Harry J. Baker, Introduction to Exceptional Children. 1948. pp. 221 - 282.

TABLE III

COMPARISON OF I.Q.'S AND ACHIEVEMENT SCORES

	SLOW						NORMAL					BRIGHT			GIFTED				
	60-64	65-69	70-74	75-79	80-84	85-89	90-94	95-99	100-104	105-109	110-114	115-119	120-124	125-130	131-134	135-139	140-144	145-149	TOTAL
95-100																1			1
90-94																		1	1
85-89											1	1		4	1	3	1		11
80-84											3	1	3	2					9
75-79							1	2	1	2	6	6	4	3	1		1		27
70-74								1		3	9	3	4	2	1				23
65-69						2	1		3	7	5	4	4			2			28
60-64							1	4	4	11	3	6	4	2					35
55-59					1	2	1	11	2	10	8	5	1						41
50-54				1	2	2	3	7	12	3	4	3	1		2				40
45-49						1		1	3	3	1	4	1	2	2				17
40-44					1	1	5	5	6	5	1	1	3	2					29
35-39				1	1	1	1	5	4	3	3	2	4						25
30-34					1	4	7	4	3	7	3	1	1						31
25-29				1		3	10	5	2	5	5	5	3	1					40
20-24					1	7	2	6	10	7	3	3	1	1	1				42
15-19					1	4	4	6	5	6	2		1	1					30
10-14				1	2	4	6	4	2		1								20
5-9	1		1		2	2	6	3		2									17
0-4				2			1		1	1	1								6
TOTAL	1		1	6	12	33	49	64	57	75	59	45	34	20	8	6	3	1	473

Note:

This table should be read as follows: Four pupils with I.Q.'s between 115 and 119 scored between 45 and 49 in the achievement test in mathematics given in Highlands County, Florida, in October, 1948.

TABLE IV

SCORES COMPARED AS TO SEX

SCORES	BOYS	GIRLS	TOTAL
0-4	2	4	6
5-9	5	12	17
10-14	6	14	20
15-19	13	17	30
20-24	19	23	42
25-29	12	19	31
30-34	22	18	40
35-39	8	17	25
40-44	11	18	29
45-49	7	10	17
50-54	18	22	40
55-59	22	19	41
60-64	18	17	35
65-69	20	8	28
70-74	12	11	23
75-79	14	13	27
80-84	5	4	9
85-89	8	3	11
90-94	1		1
95-100		1	1
TOTAL	223	250	473

Note:
This table should be read as follows: Of tewnty-seven pupils who scored between 75 and 79, 14 were boys and 13 were girls.

CHAPTER V

SIX MONTHS PROGRESS

Each course in mathematics makes a contribution toward learning how to live in our democratic society. Mathematics is concerned with problem solving; not so much in the answer, but in the method used. The psychological process used by each pupil is more important than the arithmetical answer obtained. He may not use the answer again, but he will use the method over and over. McSwain says[1] that all methods should motivate critical observation, critical thinking, and critical communication; that all classroom activities foster purposeful questioning and reflective investigation, or they retard the pupil's development in the quantitative abilities essential to intelligent citizenship.

Neither is mathematics in general education primarily concerned with vocational goals, important as they are. Rather it seeks to promote the growth of the pupil as regards reasonable competence in quantitative thinking in every day affairs,

---

1. E.T.McSwain, " Functional Programs in Arithmetic," Elementary School Journal. March, 1947. p. 383.

the disposition to use critical thinking, and the reading of the public press with discriminating intelligence. In an editorial in the Mathematics Teacher, Reeve states the following:

> It might help the present situation in secondary mathematics if we tried to give all pupils a common experience with what might be termed the absolute essentials the well-educated person should know, and then take care of individual differences in needs, interests, and abilities in two ways. First, by providing enrichment courses for superior pupils or at least for those who plan to go to college or to take up scholarly pursuits. Second, by setting up certain specialized courses for the non-college or non-academic pupils who plan to follow some special line of work.[1]

This plan might avoid terms which tend to stigmatize certain courses. Something should be done to kill the notion that general mathematics is for dull pupils only. There is a tendency to give a halo of prestige to the older, traditional courses. Mathematics teachers are selected from persons who have travelled the route of the sequential courses, and they have difficulty in teaching the newer courses with zeal, because the content and procedures are quite different. The The program needs to recognize the importance of the general courses. There is no conflict between general education and the rigorous training of future mathematicians. Both tasks

---

1. W.D.Reeve, " High School Mathematics," Mathematics Teacher. December, 1946. p. 354.

must be done well in the high school. Every pupil should be encouraged to believe that it is just as much his job to learn facts and develop skills, as it is his father's business to supply funds to finance the home or his mother's duty to keep house.

Schorling reports[1] that one out of sixteen pupils of more than 300,000 is enrolled in a second year of general mathematics, in spite of the fact that to date there is not a single textbook available specifically designed to service that course. The small school can enrich its offerings by teaching simultaneously in small classes of perhaps only a half dozen students two courses on different levels, one, general mathematics, and one algebra.

If the problems of the world are to be solved, they will be solved by intelligence, intelligence which is directed and disciplined by proper education. Therefore, the teacher is the key worker in the making of the future. The task is more than teaching facts. A physician must spend years studying facts, but merely the accumulation of knowledge does not make him a good doctor. He must know how to make use of his fund of information. Beyond science is the art of applying science even in the social adjustments of every day life.

---

1. Raleigh Schorling, "Mathematics in General Education," School Science and Mathematics. April, 1949, p. 312.

"Education is growth, but minds do not grow as bodies do, through leaving the windows open and singing in the sun",[1] is one of the favorite expressions of H.G.Wells. Spivey says[2] that the scientist, the mathematician is a hunter. He looks for facts; the facts suggest a solution. He tries the plan if he finds what the idea suggested, his plan is true.

Williamson gives seven specific work habits. She says[3] that students obtain power to organize and produce through

1. Direct attack on work
2. Prompt accomplishment of assignments
3. Accurate interpretation
4. Intelligible and adequate expression
5. Effective outlining and summarizing
6. Correct language usage
7. Neat and orderly arrangement of work

We don't make half of what we could make of our pupils. We don't make a quarter, not a tenth. They could know ever so much more, think ever so much better. Ullsvik lists[4] ten commandments for effective study of mathematics as follows:

1. Have an orderly development of new ideas
2. Pay close attention to what you are given and the kind of solution required
3. Read carefully
4. Search for uses of mathematics in daily living
5. Work for yourself rather than for the teacher

---

1. Eddy S. Kalin, "As H.G.Wells Viewed Education," The Educational Forum. March, 1949. p.384ff.
2. Dr. Ludd M. Spivey, "What is Scientific Thinking?" The School Director. March, 1949. p.6.
3. Anne E.F.Williamson, "Scholarship," Educational Administration and Supervision. January, 1949.p.25.
4. Bjarne R. Ullsvik, "Why Study Mathematics," School Science and Mathematics. April, 1949.p.278.

6. Do not be satisfied with finding only the answer. The method is more important. Your success is measured more by the method you use than by the kind of answer you find.
7. Don't use a fellow student as a crutch, for you will not have him when you come to stand on your own feet.
8. Don't be tempted in trying to fool yourself.
9. Have all necessary equipment at hand, pencil, paper, eraser.
10. No book, school, course, or teacher ever gave anyone an education; they only provide an opportunity.

The measuring stick of accuracy required in mathematics lowers the score for the student of this subject. A problem is either right or wrong. A boy gave an illustration of consecutive integers, which he had defined as " integers that follow one another in uninterrupted succession", as follows: "X, X^2, X^3", one-hundred per cent wrong. In English, this would have been one-hundred per cent right. In mathematics he must not only say the right words, but he must understand them. In other subjects he may be on the verge of understanding and still say the right thing, and thus appear to know more than he does.

Williams says:

> Give the grade teacher only a half dozen pupils, and let her work with them all day long on one process, for weeks at a time, as they did in the small instruction units in the army. At the same time lead the pupil to think that he will be shot if his attention to her wavers for a single moment, and the teacher will get results and efficiency more quickly. Along with it mental institutions will be filled up like the army hospitals.[1]

---

1. Dolphus Williams, " How Can We Improve Instruction and Achievement in Arithmetic?" Mathematics Teacher. March, 1949. p.156.

Curriculum content and methods prove their value by this yardstick, pupil growth and development. It is reasonable to expect that achievement tests given in April will show progress made during the six months following the previous October tests. There should be improvement in ability to solve verbal problems, to apply mathematical knowledge to different situations, and to think logically.

The writer has made a comparison of the progress made by those pupils in Highlands County who are taking mathematics courses with that of those who take none. The results are shown in Table V, page forty-two.

TABLE V

COMPARISON OF PROGRESS IN MATHEMATICS

OCTOBER, 1948 - APRIL, 1949

INTELLIGENCE QUOTIENT	TAKING COURSES	NOT TAKING COURSES
	AVERAGE PROGRESS	AVERAGE PROGRESS
145-149	10	6
140-144	10	6
135-139	10	6
131-134	10	6
125-130	10	6
120-124	10	6
115-119	10	6
110-114	9	7
105-109	8	3
100-104	8	3
95-99	5	2
90-94	3	1
85-89	3	1
80-84	2	1
75-79	2	1
70-74	None	None
65-69	None	None
60-64	None	None

CHAPTER VI

COMMON ERRORS

The first thing to consider in thinking about understanding in mathematics is the child. We must know something about his growth and development, his abilities, and his needs. The chief problem is the normal child who just doesn't understand arithmetic. The writer has found some who have

1. Learned by rote
2. Thought of mathematics as a bag of tricks
3. Not understood the number system
4. Seen no relationship between the processes
5. Not known that shopping for mother, taking care of an allowance, or finding out when to return a library book are all real mathematics problems.

If a pupil has depended on his memory or directions in the book, when memory fails or he does not have access to the book, he has no other resource.

We cannot assume that any junior high school child has a fund of mathematical knowledge. Each is an individual; each has had various experiences with numbers. The child must realize that arithmetic has a foundation in the past, that number is systematic; he must engage in real experiences when or where number is useful if he would have a new feeling of security and success in arithmetic.

In general, there are three types of difficulty in mathematics:

1. Lack of skill in using fundamentals
2. Difficulty with complex situations
3. Difficulty in problem solving

The writer is using fifteen kinds of examples in showing common errors. They are:

1. Simple -- requiring one operation
2. Complex -- requiring more than one operation
3. Concrete -- based on actual experiences
4. Abstract -- not touching pupil experience
5. Oral
6. Written
7. Real situation problems
8. Household problems
9. General interest problems -- money or population
10. Original problems
11. Occupational problems
12. Group problems -- several problems on one industry
13. Serial problems -- requiring a grand total
14. Problems without numbers
15. Incomplete problems

Example	Error
1. Simple	
A. Addition	
1. 29 16 8 36 100	The pupil does not know the number combinations.
2. 32 14 7 29 72	Inability to hold sixteen in mind while adding four.
3. 23 12 28 2 101	The wrong digit is carried.

Example	Error
4. 25 23 49 14 921	Both digits are written; none carried.
5. 42 19 23 17 91	One digit is discarded; none carried
6. 28 33 40 22 13 271	Addition is begun at the left column, and the digits in the last sum are reversed.
7. 28 + 32 + 15 + 10 28 22 15 10 75	The problem is copied wrong, and the total carelessly put in the wrong place.
8. Add $4\frac{1}{2}$ and $2\frac{1}{2}$ $4\frac{1}{2}$ $2\frac{1}{2}$ $6\frac{2}{4}$	Both numerators and denominators are added.

B. Subtraction

1. 3502 1317 2231	Borrowing is done correctly but forgotten.
2. 12 $10\frac{1}{3}$ $2\frac{1}{3}$	The fraction in the subtrahend is not subtracted.

Example	Error
II Complex	
A. Multiplication	
1. 4896 205 24480 4896 9792 1052640	The zero is used as one.
2. 4802 33 1446 964 11086	The zero is omitted in the partial product.
3. 4928 23 1232 8856 89792	Wrong position of second partial product.
B. Division	
1. Divide 848 by 8 .009 848/8.000 7 632 68	Confusion in meaning of dividend and divisor is shown.
2. 848 ÷ 8 $\frac{8}{848} = \frac{2}{214}$	Confusion concerning division sign is evident.
3. 16 8/848	The zero of the quotient is omitted; six is put in wrong place.
4. 27 24/640 48 170 168 2	Carelessness in subtraction is noted here.
III Concrete	
Find the cost of candy bars, at 5¢ each, when each of	

Example	Error
three children has two bars.	Poor reading comprehension

$$\begin{array}{r} 5 \\ 3 \\ \hline 15\not{c} \end{array}$$

IV. Abstract

A bridge 513 feet long was built in 8 equal divisions. How long was each division? Complete lack of thought

$$\begin{array}{r} 513 \\ \hline 8 \\ \hline 4096 \end{array}$$

V. Oral

If you come to school 18 days in a month, how many days are you absent? Calendar month is thought of instead of school month.

Child answere, "12 days."

VI. Written

1. Teacher dictates, "one hundred sixteen". Pupil writes "10016". Numbers are written in the order in which they are sounded.

2. Teacher dictates,"16". Pupil writes,"61". The six is heard first and written first.

VII. Real situation

There are 864 pupils in 12 grades. Find the average per grade. Lack of mastery of the multiplication tables.

$$12\overline{)864}\quad 80\tfrac{1}{4}$$

VIII. Household

Example: A family spends twice as much for milk as for meat. Their milk bill is $25. What is their meat bill?

25
2
$50

Error: Poor reading comprehension and carelessness.

LX. General Interest

Example: Each of America's 147,000,000 people eats 32 pounds of meat each year. How much meat do they consume in one year?

4593750
32/147000000
128
190
160
300
288
120
96
240
224
160
160

Error: Lack of judgment

X. Original

Asked to bring in an original problem after having studied fire insurance several days, one pupil submitted this one: " A man insured his house for $3000 for one year. How much did he have to pay per month?"

Error: Lack of understanding of the subject is evident. The unit must be retaught if this pupil is to profit by it. Since all the other pupils in the class may have grasped the work, the teacher will need to spend extra time with the one.

XI. Occupational

Example: You work with five other boys in a grove at 60¢ per hour. What is the payroll for all six boys for a week of six days, if all work 8 hours per day?

$$\begin{array}{r} 8 \times 6 = 48 \\ .60 \\ \hline \$288.0 \end{array}$$

Error: If this pupil had labelled his first result " hours worked by one boy", he would probably have been able to complete the problem correctly. He evidently thought that $28.80, which would have been his result had he put the decimal in the proper place, too small an amount. He therefore tried to make it large enough by manipulating the position of the decimal.

XII. Group

Example: On Monday you caught four fish that averaged 10 pounds in weight; on Tuesday you caught 3 that averaged 12 pounds; you did not fish on Wednesday; on Thursday you hooked 7 that averaged 8 pounds each. How many fish did you catch? How many pounds of fish did you catch?

The pupil raises his hand after five minutes of reading the problem over and over. He tells the teacher that the problem is so long that it confuses him. " If you would do one like it on the board, I think I could work it," he adds.

XIII. Serial

Example: A packing house processes a thousand boxes of oranges in October, three thousand boxes in November, five thousand boxes in December, and twenty thousand boxes in January. How many boxes were handled in the four months ?

1000
3000
5000
20000
29000
4
116000 boxes

Error: The pupil thought he must use every number mentioned in the problem. His thinking must be clarified.

XIV. Without numbers

Example: If you know how many cubic feet of gas you have used this month and what your bill for the month is, how will you find the cost per cubic foot?

A pupil answere, " Multiply the cubic feet by the cost."

Error: Faulty thinking or perhaps lack of thought is shown.

XV. Incomplete

Example: A man has a house worth $4500. He insures it for 5 years for $\frac{2}{3}$ of its value. What is his premium?

$\frac{2}{3} \times 4500 = \frac{9000}{3} = 30.00$
5
$150 premium

Error: The pupil did not consider carefully the facts given, or he would have discovered that something was missing. He tried to get a result that would look reasonable.

Other errors:

1. Using crutches, such as fingers for counting.
2. Counting instead of using combinations.
3. Overlooking numbers
4. Carrying when there is nothing to carry
5. Adding the same number twice.
6. Trouble with dollar sign and decimal point
7. Subtracting instead of adding
8. Failing to observe column position
9. Omitting one problem from the list
10. Adding instead of subtracting
11. Borrowing from second figure to the left
12. Forgetting one has been borrowed
13. Borrowing more than one
14. Leaving left figure dangling, i.e.
$$\begin{array}{r} 393 \\ 141 \\ \hline 18 \end{array}$$
15. Bringing down left figure that should vanish
i.e. $24\overline{)120}$ (4)
$$\begin{array}{r} 96 \\ \hline 134 \end{array}$$
16. Having zero left after taking some number from 0
i.e.
$$\begin{array}{r} 240 \\ 123 \\ \hline 120 \end{array}$$
17. Having zero left after taking 0 from a number
18. Using 15 for a whole
19. Using a fraction and 1 together

In algebra, there is often difficulty in

1. Collecting terms
2. Transposing terms
3. Stating equations
4. Changing signs
5. Interpreting meaning of exponents

A common difficulty in geometry is finding corresponding sides of congruent figures.

In trigonometry, pupils have trouble differentiating between trigonometric ratios.

There is no virtue in locating weaknesses unless something is done about them. If incorrect work is returned to the pupil to be reworked with no information given concerning the errors, in all probability the pupil will repeat the errors. Incorrect habits once formed carry over into succeeding years.

It is helpful to tabulate errors; to start with what pupils can do; to motivate; to test for general ability; to test for reading and other specific abilities; and to let the pupil know what progress is being made.

Whether errors are made because of inattention or because of misconception, the cause should be eradicated. Hassler says[1] "Teachers should use the Hueristic method --

---

1. Jasper O. Hassler, The Teaching of Secondary Mathematics. 1930. p. 149.

lead the pupil by skilful questioning to discover his errors and to find the desired knowledge hi self". When a teacher realizes that the class makes many errors, she should analyze her methods and her words. If the following problem is given as the fourth in a series of problems, the pupil may not get the proper meaning. "Separate 72 into three parts which shall be as 1, 2, and 3." It is easy for the pupil to think that the numbers 1, 2, and 3 refer to the first, second, and third problems in the group. One boy thought that the three angles of a triangle are supplementary because their sum is 180^{0}. The teacher should say what she means, and she should make accurate statements.

The errors cited in this chapter are concrete examples which the writer has observed in mathematics classes during the current year of 1948-1949. Members of these classes are from grades nine, ten, eleven, and twelve. The greatest difficulty seems to be that the pupils have not learned why they follow certain procedures.

Some of the better teachers devote time and effort to careful analysis of the strengths and weaknesses of each pupil. They then set up learning experiences specifically designed to overcome weaknesses discovered. It would seem that more teachers of this type are needed in the schools.

Change in curriculum may have to start within the teaching staffs of the secondary schools. A constant

attempt to make better the offerings of the mathematics program should always be foremost in the minds of teachers when they plan to meet the needs of high school students.

Some of the errors cited indicate that computation has been stressed much more than reasoning. It is generally acknowledged that a certain amount of drill is necessary. The things we most value in life are maintained by action, and best maintained by repeated action. The practice of telling the pupils what operation to perform, or of having some bright pupil tell the others what to do, and then all working the same problem is not conducive to individual thinking. It is the sincere belief of the writer that quality in thinking is much more important than the number of pages covered or the quantity of problems solved.

CHAPTER VII

MATHEMATICS REQUIREMENTS

To many people mathematics seems a forbidding subject. They are likely to look upon it as a compendium of mysterious processes or as an arduous task. They throw up their hands in horror when confronted with a situation which demands putting mathematics to use. Because of this antipathy, mathematics textbooks have been revised and much of the frightening material has been removed. Betz says[1] that mathematics has been removed from the curricula until there is very little that is useful or recognizable. Many of the courses are designed for amusement, and anything that might be "thought provoking" is carefully avoided. He believes that two years of general mathematics should be required of the eighty-five per cent of pupils who will not go to college, but who need mathematics for living.

Referring to the statement of Sir Oliver Lodge that the mathematical ignorance of the average educated person

---

1. Wm. Betz, " Looking Again at the Mathematical Situation," Mathematics Teacher. December, 1948. p.372

is complete and shameless, Keyser says[1] that a way to educational leadership sometimes is found by men whose innocence of mathematics is wellnigh complete and shameless. He adds that those who advocate so reducing mathematical requirements as practically to abolish the subject from the curriculum of general education probably lack just the sort of discipline which the subject they are depreciating is peculiarly qualified to give.

Needs for mathematical knowledge vary with the community, the type of work, and the individual. Those who will enter technical fields, business, or industry should have a knowledge of algebra, geometry, advanced algebra, and in many cases trigonometry. Those who anticipate doing actuarial work need the mathematics of compound interest, annuities, investment, and insurance. A basic ground work of algebra is necessary for nearly all groups of this sort. Various other groups need mathematics in various ways. Knowledge that is needed must be learned accurately and in such a way that it can be retained until needed. Boys and girls who leave school at whatever level should know the mathematics needed for their future work or jobs, and it is our obligation to see that they know it accurately and well.

---

1. Cassius J. Keyser, Mathematical Philosophy. 1922, p.315

Benz says:

> In one large state university half of the students made scores below eighth grade norms in operations with integers that involved problems in division. In other aspects the percentage below eighth grade norms was four per cent to forty per cent. On one test fourteen per cent of the college students fell below the fourth grade norms.[1]

Many other similar studies of college students and high school pupils could be cited. The lack of knowledge on the part of adults along these lines has not been so definitely established, but common observation indicates that it is as significant. Inglis states[2] that arithmetic was introduced into the secondary school sometime between 1814 and 1828. The conclusion is that it was taught on the college level because little or none of it was learned before, and that little must have been of a very elementary type.

Reeve expresses his opinion by saying:

> If high school mathematics is worth teaching, it is worth teaching in the high school where it belongs. Not <u>all</u> the mathematics offered should be required, but it should be made available, and it should be adapted to individual needs and differences among all who study it.[3]

Demiashkevich[4] quotes Pestalozzi (1746-1827) thus:

---

1. Harry E. Benz, "Arithmetic in the Senior High school." National Council of Teachers of Mathematics, <u>Sixteenth Yearbook</u>. 1941. p. 139.
2. Alexander Inglis, <u>Principles of Secondary Education</u>. 1918. p.25.
3. David Reeve, " Editorial", <u>Mathematics Teacher</u>. January, 1945. p. 37.
4. Michael Demiashkevich, <u>Philosophy of Education</u>. 1935. p. 227.

> I cannot hide from myself the fact that school instruction is for the great majority and for the lowest classes of no use at all.

Since then free public schools have been established, but public education in a democracy does not exist for the purpose of helping the genius to be a genius; neither does its mission find a limit in helping the average to be a good average. Scholars, investigators, reasoners, orators, statesmen of enduring reputation, poets, discoverers are rarely produced in the freedom of the wilderness. Youth should have the most favorable opportunity for learning all that is necessary for modern living. In most cases talents will seize opportunity, and opportunity will help talents.

The fact that a child has done inferior work in the lower grades is not a valid reason for advising him to take no mathematics after grade eight. Prediction is a complex problem. Achievement depends upon numerous variables which are difficult to control and measure. Some of these are interest, motivation, work habits, study skills, teaching methods, and content.

There has been a steady decrease in the amount of mathematics required for graduation from high school during the past twenty years. At present a good many states require only one unit. In 1931, nine states required one unit or less. Statistics for Louisiana's requirements covering the years from

1918-1937 are given by Karnes[1] as follows:

1918	$1\frac{1}{2}$ years of algebra and 1 year of geometry
1919	1 year of algebra and 1 year of geometry
1929	1 year of algebra, 1 year of geometry, and 1 year of arithmetic
1933	1 year of algebra
1937	1 year of mathematics

Karnes feels that the cause of this decline over the nation is partially due to the preparation and attitude of teachers. He quotes from a high school senior in Colorado as follows:

> Everything would be fine if we had good teachers. Subjects are dull because the teachers are dull. Even a course in mathematics would be fun and would accomplish real education if the teacher presented it in the right way.[2]

The same educator states that it was the educationist who dropped mathematics from the curriculum. The educationist knows more about the philosophy, psychology, and techniques of education for adolescents than the mathematician. On the other hand, the mathematician knows more about the subject and its potentialities, and he is the one who must take the lead of mathematics is to regain its place in the curriculum.

The aim used to be the development of scholars; the present aim is to develop well-educated citizens. It is fitting, therefore, that the complicated, unreasonable exercises and problems have been eliminated. Less memory

---

1. H.T.Karnes, " Preparation of Teachers of Secondary Mathematics," Mathematics Teacher. January, 1945. p. 3.
2. Ibid. p.4.

work is needed. Graphs are growing in importance; formulas and informal geometry are becoming more and more necessary. The function concept is also needed. In educational literature, to say that learning is functional implies that the thing learned promises to have real significance for the student in that it will make a difference to him in his behavior, including thinking, and in his command over his environment. The idea of function permeates modern mathematics and is thought by some authorities to be the concept best suited to unify instruction in the field.

Courses in senior high school mathematics have been made elective. Pupils who intend to go to college elect those courses, as do also those pupils who desire to specialize in commercial work. A few pupils will take mathematics because they like it. All pupils who intend to specialize in science will enter mathematics classes, for modern science is written in the language of mathematics. Lack of easy familiarity with higher mathematics is a formidable obstacle between ignorance and any real grasp of the modern conception of the universe, and that obstacle will continue to bar paths until the extraordinary importance of mathematical studies receives full and practical recognition.

Dr. E. P. Reinsch, Professor of Mathematics, Florida Southern College, Lakeland, Florida, believes that mathematics

is a must for everybody. He states his reason in this way:

> Mathematical thinking underlies all reasoning. The method of following hypotheses with deductive reasoning is practically the same in all fields.[1]

Mathematics is the science which draws necessary conclusions. It consists of emphatic assertions to the effect that if such a proposition is true of anything, then such and such a proposition is true of that thing. A chain of arguments is presented in which every link is important on its own account, in which there is an air of ease and lucidity throughout the discussion.

Mathematics plays an indispensable part in the study of the natural sciences. In economics, psychology, sociology, and anthropology, frequent use is made of the graphic representation of data, of statistics, and of simple algebraic formulas. Almost all students meet one or more of these fields either in the course of their formal education or later, and hence should be prepared early with the simple mathematical techniques required for their pursuit.[2] In 1945, a letter from " Somewhere Over There" to Celia E. Klotz, mathematics teacher, Port Angeles, Washington, said in part, " Mathematics classes were always so interesting,

---

1. Dr. B.F. Meinsch, Interview, April 23, 1949.
2. The Harvard University on General Education in a Free society.1946. p. 160.

and do we ever need that stuff out here !"

Florida requires one year of mathematics above grade eight and recommends at least one additional year.[1] Superintendents of twenty of the sixty-seven counties of the state report a requirement of two years of mathematics beyond. Thirty-one, including Highlands County, require only one year. Eight counties have diverse requirements, and eight counties did not reply to the questionnaire. Table six on page 63 and table seven on page 64 show the requirements throughout the state of Florida.

---

1. State Department of Education, Bulletin, Florida School Standards. 1948. Item E 7 3 p.42.

TABLE VI

COUNTY REQUIREMENTS

NAME OF COUNTY	ONE YEAR	TWO YEARS	NO REPORT	SPECIAL
Alachua		XX		
Baker			-	
Bay		XX		
Bradford		XX		
Brevard	X			
Broward		XX		
Calhoun	X			
Charlotte			-	
Citrus			-	
Clay		XX		
Collier	X			
Columbia	X			
Dade	X			
DeSoto	X			*
Dixie		XX		
Duval	X			**
Escambia	X			
Flagler	X			
Franklin	X			
Gadsden		XX		
Gilchrist	X			
Glades	X			
Gulf			-	
Hamilton			-	
Hardee		XX		
Hendry	X			
Hernando		XX		
Highlands	X			
Hillsborough			-	
Holmes	X			
Indian River	X			
Jackson				***
Jefferson		XX		
Lafayette	X			****
TOTAL	17	10	6	1

*DeSoto - Students are encouraged to take two years; 95 per cent do take two years.

**Duval - Most students take two years which is recommended as the minimum.

*** Jackson - Requirements vary. Most of the high schools require more than one year.

**** Lafayette - Three years are recommended for superior pupils.

TABLE VII

COUNTY REQUIREMENTS

NAME OF COUNTY	ONE YEAR	TWO YEARS	NO REPORT	SPECIAL
Lake		XX		*
Lee	X			
Leon	X			
Levy		XX		
Liberty		XX		**
Madison	X			
Manatee		XX		
Marion	X			
Martin	X			
Monroe		XX		
Nassau	X			
Okaloosa			-	
Okeechobee	X			***
Orange	X			
Osceola	X			
Palm Beach		XX		
Pasco		XX		
Pinellas		XX		
Polk		XX		
Putnam	X			
St.Johns	X			
St.Lucie	X			
Santa Rosa		XX		
Sarasota	X			
Seminole				****
Sumter	X			
Suwanee	X			
Taylor	X			
Union	X			
Volusia	X			
Wakulla		XX		*****
Walton		XX		
Washington		XX		
TOTAL	18	13	1	1

* Lake - The units may both be in general mathematics.
** Liberty - Students are encouraged to take more than two.
*** Okeechobee - Hope to require more in the near future.
**** Seminole - Boys, two years; girls, one year.
***** Wakulla - One more year will be added in 1950 -1951.

CHAPTER VIII

MATHEMATICS FOR CITIZENSHIP

There have not been enough studies to determine exactly what mathematics is necessary for the business of being a useful citizen, but Potter says[1] that past generations have found the need for mathematics ever increasing. In the fourteenth and fifteenth centuries, it was not thought necessary that the laymen should know the number system that every second grade pupil of today knows. Four hundred years ago, it was absolutely inconceivable that the laymen could ever learn how to multiply and divide as our ten year old children do at the present time.

Some future citizens, now in the high school, may not need much mathematics, but what they do need may be crucial to them. A certain minimum of mathematical skill and knowledge is necessary for even the most mediocre success in the world. Kinney thinks[2] we need a tabulation of the types

---

1. Mary A. Potter, " In Defense of Donald Dull," Mathematics Teacher. May, 1944. p. 197.
2. Lucien Kinney, " Criteria for Aims in Mathematics," Mathematics Teacher. October, 1943. p. 99.

of adult activities for which mathematical competence is a prerequisite. We also need a set of minimum requirements for these activities to serve as a basis for establishing standards.

Keyser believes[1] that human beings possess in some recognizable measure a sense for logic, for rigorous thinking, the mathematical faculty. To this family there corresponds a certain type of distinctly human activity in which all human beings are obliged to participate. Addition, subtraction, multiplication, and division are parts of arithmetic used daily by the average person and used more than anything else learned in the range of the whole school curriculum except reading and writing.

In South Africa, each sheep must be paid for separately. If two sticks of tobacco are worth one sheep, the native does not know how to give four sticks for two sheep. The Bank of England used notches cut in sticks as a method of bookkeeping away into the nineteenth century. The Greeks and Romans used letters of the alphabet for numbers. All these methods have been superseded by others which lend themselves more easily to arithmetical computation.

---

1. Cassius J. Keyser, Mathematical Philosophy. 1922, p.322.

Elementary mathematical knowledge can be regarded as essential in the home, and mathematics more advanced is desirable. Every housewife must plan meals, use milk from bottles of different sizes, use measuring cups and spoons, fruit jars,and gallon tins. She must buy groceries. The head of every household deals with taxes, insurance, interest, buying tickets, and borrowing. The farmer must know something of surveying, of weighing and measuring farm crops, of formulas for mixing fertilizer ingredients, and of the butter fat yield of cows. All adults should understand keeping cash accounts, computing income taxes, financing the cost of a home, providing for the future, depositing, checking, and investing money.

The biggest problem today is the business of living. To live successfully, one must meet and answer many challenging problems. A successful answer to many of these problems will depend upon a knowledge of mathematics, for the training in preciseness of statement and habits of orderly thinking emphasized in mathematics are pertinent to every phase of living. Schlauch says[1]:

> I am convinced that in every field of human endeavor mathematics can be employed to make better adjustments, greater control, and less waste.

---

1. Wm.S.Schlauch, " An Autobiography," Mathematics Teacher. November, 1948. p.201.

Everywhere we meet adults with otherwise reasonable training who are almost helpless in the presence of anything concerning numbers.

The chief objective in teaching mathematics is to train the pupil to become an intelligent member of his community, able to deal masterfully with his personal problems and with his share of community enterprise. The level of mathematical literacy required for these purposes is continually rising. Dr. B.P.Reinsch[1] Professor of Mathematics, Florida Southern College, Lakeland, Florida, calls attention to the fact that the average citizen needs much more mathematics today than he did twenty years ago. Society is at present confronted by so many social problems for which it is impossible to " know the answer " that it is especially important to acquire the steadying and careful habit of procedure which may come through the discipline of mathematical study. Good will and a warm heart are not enough to furnish the protection of life insurance; the formulas and tables of the actuary are in some ways more necessary.

Methods of the mathematician may be generalized to apply to all aspects of living. The worker in the field of mathematics learns to analyze evidence,

---

1. Dr. B. P. Reinsch, Interview. April, 1949.

distinguish facts from assumptions, recognize both the stated and unstated assumptions, and evaluate arguments. He learns to constantly reexamine the postulates which are behind his beliefs and which guide his actions. Mathematical methods furnish examples of good workmanship. Neatness, accuracy, and precision are merely attributes of " taking pains ", a thing essential to good work in any field. Intimately related to good workmanship is " thorough understanding ", also acquired by mathematical training. McCreery says[1] that mathematics provides the outlook and the means of understanding, genuine needs of even the ordinary citizen.

Throughout a long period of history, facility in the use of the mother tongue and facility in using numbers have been characteristics of the educated person, so says Hawkins[2] in a recent article. The aesthetic qualities of mathematics constitute features of high cultural value. Mathematics possesses high artistic content and rich aesthetic appeal. The laws which governed Poe in the writing of verse were based on mathematical principles, and primary among these were order, repetition, symmetry, and equality. Mathematics realizes in an extraordinary

---

1. Gene S. McCreery, " Mathematics for all Students," Mathematics Teacher. November, 1948. p.302.
2. G.E.Hawkins, " Value of Mathematics Training," School Science and Mathematics. February, 1949. p.118.

degree the fundamental principle of art, unity in variety. Mathematics rightly viewed possesses not only truth, but supreme beauty, a beauty cold and austere like that of sculpture, without appeal to any part of our weaker nature, yet sublimely pure, and capable of a stern perfection such as only the greatest art can show.[1]

Modern technology has stepped up the minimum requirements in mathematics for effective citizenship. They now stand a big step higher than fundamental operations of arithmetic, says Zant[2], while Brueckner gives [3] a comprehensive list of these requirements:

1. Methods of price labelling
2. Finding costs in a project
3. Widths in which cloth is sold
4. The sales slip
5. Reasons for price changes
6. How to compute real profit
7. Basis of credit
8. Per capita food costs of countries compared

Using Transportation Facilities

9. Parcel post and express service
10. Meaning of C.O.D.
11. Parcel post insurance
12. Commuter's and round trip tickets
13. How freight rates are computed

---

1. William Allen Neilson, Roads to Knowledge. 1937. p.234.
2. James H. Zant, " What are the Mathematics Needs of the High School?" Mathematics Teacher. February, 1949.p.76.
3. Leo J. Brueckner, " The Social Phase of Arithmetic Instruction," National Council of Teachers of Mathematics, Sixteenth Yearbook. 1941, p. 152.

Banking

14. Making deposits and using bank books
15. Savings account routine
16. Services of banks
17. Thrift by small savings
18. Check writing, endorsing, and cashing
19. Check stub records

Spending the Family Dollar

20. Patronizing the cafeteria
21. Making choices in purchases for thrift
22. Cost of cut flowers and potted plants
23. Bulb buying advantages
24. Magazine subscription savings
25. Library fines
26. Rent
27. Homemade and commercial costs compared
28. Grades and price of milk and gasoline
29. Quantity buying savings
30. Saving at sales
31. Advertisements
32. Accounts of receipts and expenditures
33. Time and cash payment plans
34. Car operation costs
35. Family budgets
36. Monthly bills and forms
37. Receipts
38. Cold storage food costs
39. Food container costs
40. Electrical device operation costs
41. Heating costs compared
42. Sizes of cans of preserved food
43. Large and small package costs compared
44. Water, gas, and electric rates
45. Cash discounts
46. Second-hand buying
47. Charge accounts
48. Out of season food costs

Miscellaneous

49. Safety practices
50. Testing seeds for germination

In Mankind in the Making, by H.G.Wells, there is this statement:

> The great body of physical science, a great deal of the essential fact of financial science, and endless social and political problems are only accessible and only thinkable to those who have had a sound training in mathematical analysis.[1]

Mathematics plays a part in accomplishing the purposes of education in American democracy. It helps attain the objective of self-realization by making the citizen capable of solving his problems of counting and calculating. It fosters human relationship by making him skilled in homemaking. It promotes economic efficiency by making him able to plan the economies of his own life; It contributes to civic responsibility by giving him defenses against propaganda.

---

1. H.G.Wells, Manking in the Making , p.97 as quoted by Leo J. Brueckner, " The Social Phase of Arithmetic Instruction ", National Council of Teachers of mathematics, Sixteenth Yearbook. 1941. p.156.

CHAPTER IX

RESPONSIBILITY OF THE SCHOOL

In 1944, the National Association of Secondary School Principals stated the following as one of the ten 'Imperative Needs of Youth' :

> All youth need to know how to purchase and use goods and services intelligently, understanding both the values received by the consumer and the economic consequences of their acts.[1]

This association places the responsibility for getting this knowledge to youth by asking:

1. Are graduates of your school able to evaluate advertising effectively?
2. Have you taught your students the value of the budget to the consumer?
3. Do your students possess mathematical skills sufficient to protect their own interests?
4. Have your students developed standards for guiding their expenditures?
5. Are your students aware of the difference between the cost of goods and services purchased for credit and the cost of those purchased for cash?
6. What do your students know about the various types of insurance?

---

1. National Association of Secondary School Principals, " Imperative Needs of Youth", Planning for American Youth. 1944, p.12.

7. Are your students acquainted with government efforts to regulate the production and to improve the quality of goods and services?[1]

It is generally conceded that the high school has the responsibility of insuring mathematical competence for the ordinary affairs of life. Betz quotes[2] Dr. A. John Hartky, Dean of School of Education, Stanford University, thus:

> Mathematics is a definite part of living and culture. We have no right to deny any student the opportunity to see this side of human culture, nor dare we cut him loose from school without the necessary mathematical tools.

If what we have is meeting the needs of some, let's keep it. If the needs of others are not met, let's attempt to meet them. McCreery says[3] that the school has an obligation to create capacities of one kind and another, and should explain to pupils the advantage that may result from them, though it recognizes that in many cases the capacities will not all be employed.

The place of mathematics in general education was discussed twenty-five hundred years ago when the Pythagoreans established the quadrivium -- arithmetic, geometry, astronomy, and music -- which was to be considered the heart of a liberal education.[4] Capable

---

1. Loc. cit.
2. Wm. Betz, "Competence", Mathematics Teacher, May, 1948. p.372.
3. Gene S. McCreery, "Mathematics for all Students in High School," Mathematics Teacher. November 1948. p.365.
4. Edward A. Cameron, "The Place of Mathematics in General Education," Mathematics Teacher. October, 1948. p.274.

students should be given mathematics up to the limit of their capacity of understanding.

It is the responsibility of the whole school from grade one through grade twelve to see that every student who is capable attains functional competence in Mathematics. Pupils are leaving school at graduation time with definite shortages in mathematical knowledge. It is the responsibility of the whole school to do something about it. Tobey says[1] that it is time to shift the emphasis from diagnosis, faultfinding, and remedial activities and direct it toward sound foundations through better teaching and more adequate and expert supervision.

A democracy must have a well and universally educated population. The school has a duty to the dull as well as to the bright student. Tartaglia, an Italian, was advised to get a vocational education because he was poor, maimed, and sickly. He became the greatest mathematician of his day. D'Alembert was also advised to secure vocational training and prepare for the shop or lower positions of trade. He became one of the greatest mathematicians of France.

An extended curriculum in mathematics is needed so that there may be opportunity for consideration of the many

---

1. Wm. S. Tobey, " Why Mechanical Manipulation", Mathematics Teacher. April, 1948. p. 216.

social and consumer issues. Only the superior student has a mastery of computation by the end of grade eight. To build the mastery needed for vocational requirements as well as for quantitative interpretations will require at least one year, and for half the pupils two years beyond grade eight. To build an ability to recognize aspects of social problems and to select crucial data which will lead to wise interpretation will take another year. Not all pupils could cover this work at the same rate. Potential engineers and scientists could master it in one semester after grade eight. The main thing is to have a curriculum based on the needs of every pupil and to see that each one has the opportunity to master the necessary skills whether it take one semester or three years.

Mathematics is the very embodiment of permanence. It does not emerge from day to day; it is rooted in a very deep soil. Long after the vagaries of this age shall have been swept away, mathematics will continue to be one of the great unifiers of the race, because of its global significance and its cosmic perspective. If there be a third world war, it will be a mathematical war. In war or in peace, neglect of mathematics in the schools a case of killing the goose that lays the golden egg.

We cannot afford not to train our potential leaders, neither can we escape the fact that citizenship in a new

world demands that all be trained at their level of ability to comprehend the basic mathematical concepts of our social and economic problems. If a student's vocation has been determined, he should know its mathematics requirements. Peters says[1] that the school should provide cheerful rooms, available materials, a mathematics laboratory, a library with histories of mathematics, popular editions of mathematics magazines, biographies of mathematicians, and cases for exhibits and geometric figures.

Nor can we afford to leave mathematical training to the caprice of the students. An assumption of the elective system is that the student is himself competent to fix the pattern of his education. The difficulty with this assumption is that it attributes to the student at the beginning a wisdom that comes to him only at the end of his high-school years, if indeed even then. To leave a child free to study any subject or none is simply to deprive him of his social inheritance.

Reflective thinking stands at the heart of the method of scientific inquiry. The study of mathematics is of educational value because mathematics can be made to throw the problem solving process into sharp relief, and so offers

---

1. Ann C. Peters, "Helpful Guidance in the Teaching of Mathematics," Mathematics Teacher. December, 1946. p.353.

opportunity to improve thinking in all fields. Douglass says[1] it has been found that the average mark in high school mathematics has a definite correlation with the average college mark in every field.

Thinking applied to the real problems in the basic relationships of living provides a valuable tool for the preservation of the democratic way of life. Students who are in the habit of formulating real problems and insisting on genuine solutions, who know how to judge, collect, and interpret data, and who know how to recognize valid proof, will not be so easily led by propaganda.

Fractions, equations, congruence, similarity, area, function, logical implication, group properties, and quantitative variation are involved in dealing with mathematical questions that arise in analyzing many problems, and in facing many situations,and are important in the solution of every problem whether or not it calls for characteristically mathematical treatment. Mathematics has something to contribute to the meeting of needs in every aspect of living. The ability to interpret data expressed in graphical form is useful in relation to needs in each area whether (1) personal living, (2) social relationships, (3) social civic relationships, or (4) economic relationships.

H.R.Douglass," The Relation of High School Mathematics to College Marks," School Review, March, 1936. p.617.

Without algebra and geometry no aviation, power transmission, modern bridges, tunnels, or skyscrapers would be possible. Without formulas, there would be no chemistry, physics, nor astronomy. Mathematics is only a part, but a very essential part, of the needs of all students. A strong and corrective influence should be exerted upon those boys and girls who are capable of doing fair work with secondary mathematics, but who take what is easiest or most glamorous at the moment.

The interest of average pupils and even of some of the stronger ones may be stimulated by teaching methods used in the business world. An understanding of machine calculation, which has greatly simplified numerical computation, may increase a pupil's opportunity of finding employment. The school should provide low priced machines which are designed especially for school use. Transits and sextants are instructive instruments that should be in mathematics classrooms. When more and better films having a mathematical bearing are developed, they will be found most interesting and useful in the schools.

It should be the special duty of the high school to see that every graduate has a sufficient knowledge of mathematics to enable him to take his place in his community, and not be embarrassed in ordinary mathematical

situations that are bound to arise. The school should make the idea of scholarship more honorable and attractive. Scholars should have public respect. They are dreamers, but only the small bits of their dreams that come to pass make life endurable for the rest of the world.

It is for the good of society that each year there should go forth from the secondary schools a large number of young people with marked proficiency in the technical skills of mathematics, together with a fundamental understanding of some of its concepts. The steady flow of such a group into our democracy is a major responsibility of school administrators.

CHAPTER X

A TEACHER. WHY?

To be a teacher one must be a scholar, a practical person, and an artist. The teacher must know children, for he bears the responsibility of equipping them to solve the actual problems of living. The mathematics teacher must make mathematics meaningful, thrilling, alive, satisfying, and always within the comprehension of the pupil. It is the teacher who must develop in the pupil honesty, industry, ability to concentrate, neatness, preciseness, self-confidence, thoroughness, and a desire to know and to improve.

It is imperative for the teacher to consider the end product of teaching. Mathematics is a precise method of thought. If precise facts are fed into it, precise results are obtained; if approximate facts, the results will be approximate. The solution of problems is not unique in mathematics. Manipulation of numbers is only an aid in setting down thoughts clearly and concisely. In verbal problems one reads between the lines; in life one

imagines conditions. In problems one recognizes the real problem which is often hidden; this is true in life. In problems one analyzes the best approach from the many possible ones; this is more true in life. In problems the parts that have no bearing on the solution are recognized, and no time is wasted with them; this is very hard to do in life. In solving problems one must anticipate and estimate what the possible solutions may be; this is also true in life. Mathematics is beneficial because its symbolism enables one to isolate attitudes and facts.

A teacher can teach mathematics only if he knows the subject and if he is a continual student of it. He must have meaning, understanding, and reality. Book problems may seem real to the author, but the teacher must breathe the breath of life into them for the pupils. Fehr believes[1] that real teachers of mathematics like to teach; they enjoy giving knowledge to others; they like to develop a spirit of inquiry, of search for truth; they like the challenge mathematics gives to correct and vigorous thinking; they thrill at its proofs and never tire of it.

The mathematics teacher's complexities seem to have varied directly as the increase of the school population.

---

1. Howard H. Fehr, " Teachers of Mathematics," Mathematics Teacher. October, 1948. p. 370.

He has direct charge of the pupils, interprets standards or makes his own standards of achievement, prepares, administers, and scores tests, interprets and applies the curriculum, diagnoses pupil difficulties, uses remedial measures, adapts instruction to individual pupils, and provides curricular enrichments for superior pupils. The educational experiences which children have in school are controlled largely by the teacher.

Homogeneous groups within a class require a master teacher, resourceful, a good organizer, a good disciplinarian with tact to make simple tasks so enticing that pupils may get a thrill from worthy achievement. Sometimes there is a class of slow pupils whose powers of association are low, who do not see relationships easily, who are lacking in social intelligence, who have physical defects, who are emotionally unstable, excitable, repressed, or lethargic. It is amazing to see what a patient teacher, who is prepared to make full allowance for these children and to plan a syllabus along appropriate lines, can extract from this unpromising material. The teacher's challenge is to correct errors of understanding and introduce social situations in such a way that pupils will live them. When there are rapid learners in the class, the challenge is to provide sufficient worthwhile materials for them.

Breadth of outlook and understanding are essential qualifications of a good citizen, and, therefore, even more indispensable qualities for prospective teachers who will be expected to play an important part in the development of good citizens. If mathematics is to be taught with lifelike situations in view, then it will be necessary for teachers of mathematics to have an appreciation of its varied applications. Prospective teachers of secondary mathematics should contact as many of the broad fields of learning as possible. They should have work in physical science, biological science, social science, literature, art, and languages. Karnes says[1] that a mathematics teacher is a seller of mathematics. As such he must pay attention to the background, foundations, and uses of his chosen field.

No longer is the student body a select group of college-bound students, but pupils who vary a great deal in interests and intelligence. Instruction and guidance are inseparable twins. This puts a heavy responsibility upon the teacher who must be prepared not only from the subject matter point of view, but also from the point of view of fundamental educational techniques and principles

---

1. H.T.Karnes, " Preparation of Teachers of Secondary Mathematics," Mathematics Teacher. January, 1945. p.6.

in order to give instruction, guidance, and service to the heterogeneous group of pupils in the school.

The teacher should commend economy, discourage useless extravagance of detail, and make clear what is meant by neatness of proof, its clarity, directness, cleverness, and economy. If there are two possible formulations of a problem, that which leads to the more general result is usually to be preferred. The mathematics teacher must build his program toward an affirmative answer to such questions as the following:

1. Does it help adolescents meet their needs in the major aspects of living?
2. Does it promote, enrich, and refine democratic ideals through the development of related desirable qualities of personality?

No teacher will attain all his goals, but Peavy says[1] that if some improvement is evident, the work has not been wasted.

Schorling[2] lists twelve factors which cut down the efficiency of the mathematics teacher:

1. Extracurricular duties
2. Large classes
3. Being compelled to work at an out-of-school job to supplement income
4. Lack of equipment
5. Lack of supplementary materials
6. No permanent classroom
7. Too many preparations

---

1. Katherine S. Peavy, " Arithmetic and Life Experiences," Grade Teacher. March, 1948. p.150.

8. Being required to teach unrelated courses
9. Worry about moderate income
10. Outdated textbooks
11. Lack of right kind of supervision
12. Inadequate training for the job

Traxler enumerates[1] seven objectives of mathematics teaching as follows:

1. Skill in the fundamental operations
2. Ability to recognize mathematical relationships
3. Understanding of mathematical concepts
4. Ability to solve verbal problems
5. Ability to apply mathematical knowledge to new situations
6. Ability and habit of making precise statements and thinking logically
7. Understanding and appreciation of the contribution of mathematics to civilization

The writer makes the following suggestions as necessary tasks for mathematics teachers:

1. Correct faulty methods
2. Teach checking
3. Use good pupils to assist others
4. Show pupils their improvement
5. Provide for variation in rate of progress
6. Set up standards of attainment
7. Select materials of proper level of difficulty
8. Make pupils conscious of difficulties
9. Help pupils diagnose difficulties
10. Help pupils overcome difficulties
11. Provide opportunity for participation in class activities
12. Supervise the work
13. Require neatness

Mathematics is second to no other subject in high school, but we need additional emphasis upon accuracy and efficiency

---

1. Arthur E. Traxler, "Testing in Mathematics at Secondary Level," Educational Records Bureau. 1939. p.21. as quoted by Harold Alberty, " Reorganizing the High School Curriculum. 1948. p.378.

in mathematics teaching. Teachers should select the most promising students and influence them to become teachers.

The writer lists some techniques considered desirable for mathematics teachers to follow:

1. Provide conditions in which growth of desirable social characteristics, attitudes, and ideals will take place.
2. Recognize arithmetic applications in other subjects, such as history, geography, and science
3. Bring experiences and activities in connection with local conditions and situations into class.

The mathematics taught must start from real and interesting situations if we are to expect students to be sensible about it. The desired one hundred per cent mastery ideal may be obtained only by careful analysis of methods used by the pupils, by a pupil-directed program of improvement, by giving remedial instruction in the fundamental processes, and by much explanation. Teachers of mathematics should be at the loss of popularity of mathematics courses in the high school and college at a time when the world is pretty well convinced that it is mathematics and science that make the world go around.

The mathematics teacher should know more than he can teach pupils, and he should continue to learn old and new methods and ideas of arrangement. He should study the history of the development of the subject. He should maintain membership in an organization of mathematics teachers. He should develop his own methods and make

himself skillful in many incidental operations. He should be able to teach without the book. He should make sure that pupils understand. He should make sure of mechanical efficiency. He should relate the subject to practical applications, and he should develop independent thinkers.

Good will is the measure of the teacher's success and determines his usefulness to the school and the community. Teacher support, loyalty, and enthusiasm for the school are most contagious and will result in increased community support, loyalty, and enthusiasm.

Every mathematics teacher should ask himself the question, " How good a job am I doing in providing young people with an intelligent basis for choosing mathematics as a profession?" The teacher is aware that his knowledge is only a small tower from the top of which he may survey ever larger portions of the vistas of human knowledge. The higher he builds his own tower, the more distinct the horizons he can perceive. If he teaches right, he makes an important contribution to the totality of knowledge and thought, to the progress of civilization, and to human happiness.

CHAPTER XI

CONCLUSIONS

Education has been primarily concerned with making people literate. Book learning that developed abilities to memorize, to read, to comprehend, and to recite has been of fundamental importance. The general philosophy of education must be enlarged. Education must be recognized as a continuous process from birth to death. It is not something that goes on a few hours a day within the walls of a school. It arises from, exists in, and will continue to flourish as a part of its surrounding social culture. It cannot be completely isolated from life outside school.

Arithmetic is the one school subject that is full of logical relationships that facilitate learning and give meaning to it. There are three types or levels of learning that may be anticipated by the teacher of mathematics:

1. Learning for permanent mastery computations which all people need in the conduct of daily affairs. As a definite aid in the mastery of facts or in the perfecting of the steps in a process, a type of practice is recommended which includes practice in thinking and practice in seeing

relationships. Ideally, every pupil should advance at his own rate of learning.

2. Learning for temporary understanding, lesser used facts of measurement and semi-technical computations easily forgotten.

3. Learning for partial understanding and appreciation, foreign currency and the way to find the rate of interest in installment buying problems. Only the more able pupils might be expected to participate in this type of learning, while all pupils might be expected to learn to add and to know when to add to a high degree of perfection.

That arithmetic made its first appearance as a college requirement in 1745 is reported by Kandel,[1] and he refers to it as 'common' arithmetic. The curriculum of the American elementary school down to the Revolution included reading and writing as the fundamental subjects, with perhaps a little arithmetic for the more favored schools. Eventually it disappeared from the high school and became one of the basic subjects in the elementary school. The proposal here made is that arithmetic be given more attention as a high school subject.

---

1. I.L.Kandel, History of Secondary Education. 1918. p.128 as quoted by Harry E. Benz, " Arithmetic in the Senior High School," National Council of Teachers of Mathematics, Sixteenth Yearbook. 1941. p.134.

Arithmetic cannot be imposed upon a pupil, but must be developed by his own individual responses and reactions. Neither can arithmetic be left to individual caprice, but must be made a subject of explicit instruction. Distinguished scholars connected with higher educational institutions are interested in reform of mathematic instruction. Their suggestions include emphasis on functional thinking or the study of relationships, and genuine acquaintance with key concepts and methods.

Betz lists[1] three main components of functional competence in mathematics as follows:

1. Systematic study of underlying mathematical concepts, principles, skills, and modes of thinking
2. Proper emphasis on the significant interrelationships between mathematical theory and its many-sided applications
3. Emphasis on understanding and mastery.

He tells how to attain functional competence by (1) giving attention to the three components listed, (2) building continuous curricula, (3) rejecting automatic promotion, (4) having a two track program, (5) insisting on more adequate training programs for those entering the profession.

If the present low level of mastery of the fundamentals of arithmetic manifested in the senior high school

---

1. Wm. Betz, "Competence in Mathematics," Mathematics Teacher. May, 1948. p. 321.

is not sufficient, how shall the courses of the senior high school be reorganized to build up the skills? Is the child taught what he will not have use for until he has forgotten it? Should taxation, banking, and investment be taught in grade eight or nine to boys and girls of thirteen and fourteen years of age? Eighth grade boys have no immediate thought of buying a car, neither are girls of that age interested in family projects. Junior and senior students are both interested in and sufficiently mature to appreciate the full significance of the cost of supporting a family or the effect of socialized medicine. However, these courses are, in the field of mathematics, usually elective and taken only by the bright, ambitious pupils who constitute a small proportion of the group of boys and girls in the school.

In many cases eighth graders could do better on an arithmetic achievement test than seniors. Some high schools give an arithmetic test to all seniors to find those who cannot solve problems with which an adult is faced. These pupils are given special teaching in certain periods by mathematics teachers in addition to their regular work. Brueckner strongly recommends[1] for such work the selection

---

1. Leo J. Brueckner, " The Social Phase of Arithmetic Instruction," National Council of Teachers of Mathematics, Sixteenth Yearbook, 1941. p.144.

of suitable and significant topics dealing with social processes in which number functions directly as a basis of units of instruction.

The poverty of the results of arithmetic teaching has been exposed in both pupils and adults who are not able to solve quantitative problems and are afraid to face any situation involving numbers. Brownell believes[1] that if teachers are to have the information they need, research must be given a new direction. The new point of view emphasizes relatedness rather than itemization. It stresses generalization instead of extreme specificity. It conceives of learning as a meaningful, not a mechanical process. It considers understanding more important than mere repetition. It looks upon learning as a developmental process, not one of fixation of stereotyped reactions. It encourages discovery and problem solving, rather than rote learning and parrot-like repetition.

Educational objectives will center around three things, namely, the physical world, the social world, and the individual. Every school activity of importance has both an impersonal and a personal aspect. The impersonal

---

1. Wm.A. Brownell, " The Evaluation of Learning of Arithmetic," National Council of Teachers of Mathematics, Sixteenth Yearbook.1941. p.266.

aspect deals with the developing experience of the race; the personal, with the reactions of the individual. It is hoped that the interpretations of the findings of this study will be useful in an expansion of the scope of secondary mathematics training in the interest of a richer program.

A three-year mathematics requirement could be set up. This would gather into the sequential classes those able pupils who do not elect such subjects because "they are too hard". It would also guarantee to terminal pupils sufficient mathematical competence to carry on the ordinary affairs of life. Research evidence indicates a positive relationship between achievement in mathematics and the number of such courses taken.

Benz calls attention[1] to certain possibilities with reference to the inclusion of material commonly called arithmetic in the program of the senior high school , and he illustrates various types of material which are available:

1. Set up a new course in senior high school arithmetic and present the material desired through the medium of activities which transcend the usual subject boundaries (possibly the core curriculum)

2. Use applied arithmetic as follows:

---

1. Harry E. Benz, "Mathematics in the Senior High School," National Council of Teachers of Mathematics, Sixteenth Yearbook. 1941. p. 123.

A. Taxes
1. Kinds
2. Just what is taxed by each kind
3. Who finally pays each kind of tax
4. Passing taxes on to the consumer
5. How taxes pyramid
6. Taxing various groups, consumers, manufacturers, motorists, rich, poor, middlemen
7. Political implications in taxes. Why it is relatively easy to tax many people small amounts
8. Tax collecting machinery, state, federal, local
9. Tax limitation laws; their value
10. Taxes as fees for services
11. Taxing frugality, thrift, enterprise, inventiveness
12. How tax money is spent, a study of governmental budgets
13. Taxes which save money, new fire fighting equipment may reduce the insurance premium

Such materials should be included in a course which might be called "General Education". It could incorporate business letter writing, stress spelling, and give art appreciation, as well as other materials that would interest students sixteen to eighteen years of age. A name for the course, other than arithmetic, might make it more attractive to these students.

The fine arts, including music and literature have provided elements of release from drudgery and sources of appreciation that are among humanity's noblest treasures. To permit continuous contacts with these great and enduring products of human genius would seem to be a sacred obligation of the school. Situated at a still higher level there is a reservoir of spiritual values and visions pertaining to a

nobler and finer life, to ideals of heroism, and self-sacrificing endeavor. Such values may not be suitable for textbooks and courses of study, but it is not too much to expect that the school shall, by its spirit and its daily activities, keep in mind the infinitely subtle and permanent problem of personality growth.

BIBLIOGRAPHY

Alberty, Harold, Reorganizing the High School Curriculum. New York: The Macmillan Company, 1947. p.X 458

Butler, Charles Henry, and Wren, F.L., The Teaching of Secondary Mathematics. New York: McGraw-Hill Book Company, 1941. p.XII 514.

Curry, Preston W., and Rice, Ralph R., Applied Business Arithmetic. New York: South-Western Publishing Company. 1945.p.II 576.

Demiashkevich, Michael, An Introduction to Philosophy of Education. New York: Houghton, Mifflin Company, 1918. p.II 215.

Harvard University Committee on General Education in a Free Society. Cambridge: Harvard University Press, 1946. p.XIX 267.

Hassler, Jasper O., The Teaching of Secondary Mathematics. New York: The Macmillan Company, 1930. p. XI 405

Hogben, Lancelot, Mathematics for the Million. New York: W.W.Norton and Company, 1937. p.V 837.

Hooper, A., A Mathematics Refresher. New York: Henry Holt and Company, 1946. p.VIII 361.

Hooper, A., An Arithmetic Refresher. New York: Henry Holt and Company, 1944. p. VIII 119.

Inglis, Alexander, Principles of Secondary Education. New York: Houghton, Mifflin Company, 1918. p. XI 215.

Keyser, Cassius J., Mathematical Philosophy
New York: E.PDutton and Company, Incorporated,
1922. p.XIV 466

Kimball, Reginald Stevens, Practical Mathematics.
New York: National Education Alliance, 1947. p.[illegible] 320.

National Council of Teachers of Mathematics,
Fourth Yearbook. 1929. p.X 186.

National Council of Teachers of Mathematics,
Fifteenth Yearbook. 1940. p.XVI 253.

National Council of Teachers of Mathematics,
Sixteenth Yearbook. p.XII 335.

Neilson, William A. Roads to Knowledge. New York:
W.W.Norton and Company, 1937. p.VIII 419.

Parker, S.C., The History of Modern Elementary
Education. Boston: Ginn and Company, 1912. p.XXIV 419.

Elementary School Journal. Department of Education. University of Chicago.

Grade Teacher

Journal of Higher Education. Bureau of Research. University of Ohio.

Journal of Exceptional Children. International Council for Exceptional Children.

Mathematics

Mathematics Teacher. Official Journal of Teachers of Mathematics.

National Educational Journal

School Review. Department of Education. University of Chicago.

School Science and Mathematics.

The American Scholar. Phi Beta Kappa.